A NEW ELECTRODYNAMIC METHOD OF MEASURING MAGNETIC FIELDS

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(*Received for publication, March 3, 1954*)

Plate V

ABSTRACT. A new electrodynamic method has been developed for the measurement of magnetic fields. The new method is based upon the measurement of the maximum couple exerted upon a small current-bearing coil suitably suspended with a fine torsion fibre in the magnetic field. The method is very simple, quick and with reasonable precautions gives an absolute field accuracy of the same order as standard ballistic or electrodynamic methods. In this paper the details of the method are described and results of several field values are compared with those from search coil and crystal methods.

INTRODUCTION

In a few recent papers Bose and Mitra (unpublished) have found that the anisotropies of the paramagnetic ions Cu^{++} and Ni^{++} change from salt to salt in the isomorphous Tutton series approximately in accordance with alkali cation radius. These observations have been attributed mainly to the variation of the long range asymmetric crystalline electric fields (direct and indirect) from salt to salt. Since the observed changes in the anisotropies are small and their accuracy depends to a large extent on the room temperature values measured by Krishnan et al (1933-38) it is necessary at this stage to check and improve upon these earlier values. The method of measurement of magnetic anisotropy of single crystals, of which details need not be given, depends upon the measurement of the maximum couple exerted upon a crystal suspended vertically in a homogeneous horizontal magnetic field H, with a fine quartz fibre from a torsion head. The anisotropy ΔX in the horizontal plane is given by the general equation

$$c(\theta-\phi) = \frac{2m}{M} \Delta X H^2 \sin 2\phi \qquad (1)$$

where m is the mass of the crystal, M its molecular weight, H the field strength, c the torsion constant of the fibre, θ the torsion angle of the fibre and ϕ the angle which the direction of the maximum susceptibility of the crystal in the horizontal plane makes with the field direction.

In pursuance of our present programme of improving the accuracy of anisotropy measurements to 0.1% or better, the accurate determination of the torsion constant c was undertaken by Datta (1953). Since the square of the magnetic field comes in the equation, it is also necessary to determine this with a high degree of accuracy. The values of the field by the earlier workers in this laboratory were obtained by the usual search coil methed of which the accuracy was not probably more than about 1 to ½%. A crystal itself may be used for simple and quick field measurement provided its anisotropy has been measured in a well standardised field Later on, it has been the practice in this laboratory to use for this purpose a crystal standardised by the ballistic method and checked by the Gouy method using $NiCl_2$ solution. A crystal gaussmeter has been devised by Dupouy (1951) following the same principle. But evidently, the accuracy of the crystal method depends ultimately on that of the primary method used The bismuth spiral method (Bates, 1951), though sometimes convenient is also a secondary method of only moderate accuracy. Methods dependent on the rotating search coil, Zeeman effect, β-ray spectrometry, proton resonance, etc. (Bates, *loc. cit.*), though of great accuracy, are not available to every laboratory and are suitable only for specialised purposes. Absolute electro-dynamic methods, depending upon the measurement of translational force upon a current-bearing conductor placed in the magnetic field, have been developed by Cotton, Piccard and Devaud (1932) and by Briggs and Harper (1936) and though the accuracy claimed are as high as .02 to .1 per cent, the arrangements are rather cumbrous and cannot be used for exploration of fields. Klopsteg (1913) has utilised the damping of a galvanometer coil suspended in a magnetic field to measure it. But the method though ingeneous cannot claim an accuracy of more than 1 or 2%.

It occurred to us to use Klopsteg's experimental system in a manner exactly analogous to our crystal anisotropy method for measuring the magnetic field. The method is simple, quick and with reasonable precautions capable of giving an absolute field accuracy of the same order as standard ballistic (1939) or electrodynamic methods.

THEORY

For a coil of n turns and mean effective area A, carrying a current i suspended in a homogeneous magnetic field H with a fibre of small torsion constant c, the equilibrium condition at the angle ϕ which the normal to the plane of the coil makes with the direction of the field, is given by

$$c(\theta-\phi) = niAH \sin \phi \qquad \ldots \quad (2)$$

where θ is the torsion angle of the fibre.

Starting with the plane of the coil at right angles to the field and no torque on the fibre, *i.e.*, $\phi = 0°$ and $\theta = 0°$, if the fibre is gradually twisted the plane of the coil will follow but through a smaller angle until the position of maximum couple due to field ($\phi = 30°$) is reached, For θ very large compared to ϕ this is also an unstable equilibrium position of the coil, which will now sharply spin round with the slightest increase of the torsion angle or with little disturbance. In this critical position which can be accurately marked

$$H = \frac{c(\theta_c - \pi/2)}{nAi} \qquad \dots \quad (3)$$

from which the field can be accurately measured. This is an exact electro-dynamic analogue of the crystal method so long used by us. For small values of θ_c, a correction might be made as obtained from the condition of instability in eqn (2), thus

$$H = \frac{c}{nAi} \cdot \frac{(\theta_c - \pi/2 - \sigma)}{\cos \sigma} \qquad \dots \quad (4)$$

$$\text{where } \sin \sigma = \frac{c}{iNAH}$$

But the procedure is then not capable of high accuracy. As we could not procure very fine phosphor-bronze strips or metal-coated quartz fibre for suspension of the coil, we had to use fine silver strips, which did not stand large torsions without slight yielding, and so our critical angles of torsion could not be made large. Hence in practice the above method could not be used profitably.

A slightly different procedure, avoiding the correction factor, was to find the maximum couple in the field by adopting a null method which though somewhat more elaborate is particularly suitable for small angles of torsion and has one obvious practical advantage over the previous method, that the final observation here is for a static position. The coil is placed in the maximum torque position in the field, with no torsion on the fibre in the absence of the field (*i.e.* $\phi = 90^\circ$, $\theta = 0^\circ$). The field is put on and the coil is deflected, when it is brought back to the original position by twisting the fibre. The simpler equation

$$H = \frac{c\,\theta_{\text{max.}}}{niA} \qquad \dots \quad (5)$$

then holds for any value of the maximum angle of torsion.

PREPARATION OF THE COIL

The preparation of the coil is the most important part in the construction of the apparatus. Very great care is necessary in selecting the material of the former, on which the coil is wound. Ebonite samples had to be rejected as most of them showed a stray tendency of setting in the magnetic field possibly owing to ferromagnetic impurities. Several kinds of insulating paper samples also were rejected for the same reason. Very thin-walled (about 0.1 mm) pyrex tube of about 1.5 to 1 cm. diameter and of the same length was found to be very suitable for the purpose and in a field of about

5000 gauss showed no setting tendency. The former was thoroughly cleaned and then coated with specially prepared shellac varnish, One layer of fine double silk-covered copper wire was closely and accurately wound on the former mounted on a small jeweller's lathe, keeping a constant tension on the wire. The coil was soaked with thin shellac varnish and dried several times and finally baked for several hours in an air-oven below 100°C. The first and last windings were fixed firmly with Durofix cement. Procautions were taken to avoid dust as far as possible during these procedures. The two free ends of the copper wire passed through short lengths of fine pyrex capillary tubes, attached diametrically at the opposite sides of the spool with pure shellac or Durofix cement. After this it was again checked for any setting tendency due to possible impurities in the wire and shellac varnish. Several coils were prepared some of about 1 cm. diameter and some 1.5 cms. The smaller ones were wound with 60 s.w.g. and the larger ones with 44 s.w.g. wire. Two very light small triangular mirrors prepared from microscope cover slips were attached at right angles to each other on the top of the coil, axially to the suspension fibre.

SUSPENSION SYSTEM

The entire suspended rotating coil system is shown in Plate V. The suspension fibre was a fine silver wire .001 mm. diameter and about 40 cms. long. With fine phosphor bronze or metal-coated quartz fibre the results can be easily improved. The silver wire was carefully selected to avoid kinks or non-uniformities, and allowed to hang under a sufficient load and heated by an electric current to dull red heat to make it straight otherwise serious uncertainties in the results might occur. One end of the fibre is soldered, using a microjet and specially prepared tin solder and diamagnetic resin flux, axially to the adjustable brass pin of a torsion head, with a vernier reading to 1/10 of a degree and an insulating ebonite knob for turning. The free end of the torsion fibre was soldered to the upper terminal of the coil and a silver helical spring of very small torsion constant to the lower terminal. To prevent draughts disturbing the system, it was enclosed in a glass tube, with plane front and back windows for viewing the reflecting mirror. The free spring end was soldered to the adjustable brass pin of another small torsion head, with an ebonite turning knob, fitted to the lower end of the enclosing glass tube. The two torsion heads and the entire suspension system were made coaxial with the greatest precaution. Special care was taken to see that the plane of the coil was vertical and the suspension fibre along the central vertical diameter of the coil.

MODE OF MEASUREMENT

(i) *Measurement of Radius*:

The coil radius was ascertained very accurately with the help of a comparator reading to 1/1000 mm. At a particular point on the axis of the

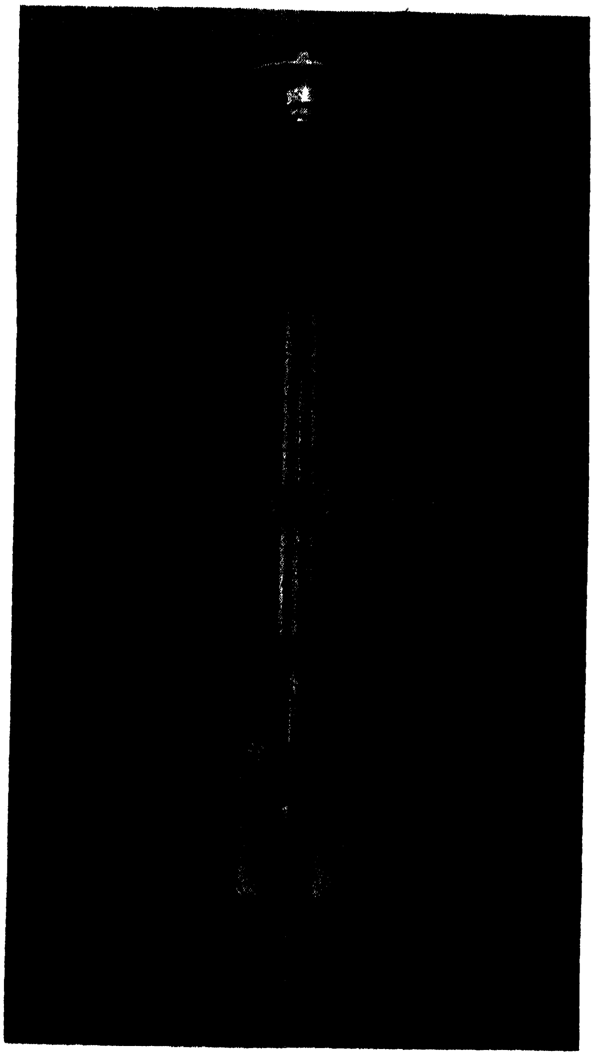

Fig. 1.

Photograph of the experimental arrangement

a. Torsion head
b. Ebonite turning knob
c. Torsion fibre
d. Coil
e. Mirror
f. Spring
g. Clamp
h. Lower torsion head
m. Glass case
l. Window
n, Magnet

glass tube the two diameters before and after winding were measured at a large number of positions along the circumference and the mean taken from which, allowing for the insulation thickness, the effective area was calculated. A small difference in the area not more than ½% and changing linearlly to within .0001 cm. was found to occur along the entire axis of the glass tube. So, taking the mean of as many readings along the axis as there were number of turns we were able to find the mean effective area of the entire coil with a high degree of accuracy.

(ii) Measurement of Torsion Constant:

Before attaching to the coil, the torsion constant of the fibre was measured very accurately following the procedures described by Datta (*l. c.*) using one of his standard glass discs, suspended from the fibre with its plane vertical and noting the time period of the system with a stop watch reading to 1/10 sec. and checked periodically against a standard clock.

(iii) Measurement of Current in the Coil:

The current in the coil was of the order of microamperes and was taken from a single lead storage cell of large capacity through a megohm range rheostat. The actual values of the current could be measured for ordinary use with a Weston microammeter carefully calibrated potentiometrically against a L & N standard 100 ohm resistance, kindly lent to us by Dr. P. C. Mahanti of the Applied Physics Laboratory, Calcutta University. But for standardization work the currents were directly measured from the fall of potential across the standard 100 ohm resistance with a L & N Dial type potentiometer reading 5 microvolts per small division. With the very feeble currents used in the coil, about 25 to 70 microamperes, the heating effect was negligible.

(iv) Measurement of Maximum Angles of Torsion in Magnetic Fields:

The apparatus was arrangsd so that the coil was placed centrally between the parallel square pole pieces (4″ × 4″ × pole gap 1½″) of an electromagnet which we usually use for magnetic anisotropy measurements of crystals at room temperature. On exciting the coil and the magnet simultaneously the plane of the coil tended to set at right angles to the field and by releasing the torsion on the fibre and the spring by rotating both the upper and the lower torsion heads, the coil set perfectly and showed no movement when the current in the coil was switched on or off. This condition could be accurately brought about by observing through a telescope the image of an illuminated scale upon one or the other of the two mirrors attached to the coil. Currents were then switched off and both the torsion heads were rotated through 90° in the same direction to bring the plane of the coil along the direction of the field, corresponding to the position of maximum magnetic couple on the coil which was noted carefully in the eye piece scale of the telescope. Any residual inaccuracy in this position of initial setting could be easily eliminated by reversing the magnetic field, but not the coil current, obtaining the corresponding setting position beginning from the opposite side and taking the mean of the two observations if a slight difference was observed, which was rare.

When this had been accurately achieved the magnetic field was reversed several times to obtain a steady magnetic state, taking care to switch off the current in the suspended coil. At this juncture, before switching on finally the magnet-current the residual field could be measured and its constancy for different experiments checked by balancing the deflection of the coil, when a known current was sent through it, by turning the torsion head. A current between 25 to 70 microamperes was then allowed to flow through the coil. The coil was deflected and the torsion head was rotated to bring the coil accurately back to its initial setting as observed by the telescope and scale arrangement. We have not used any special device for automatically stabilizing the magnet current except that we have taken the current from a 10 kilowatt compound wound D.C. generator instead of directly from our 6 phase rectified D.C. mains which fluctuates rather badly and also relied upon the high inductance of the magnet to stabilise the current further. At any rate the accurate balance position of the coil (a static position) for a given current could always be adjusted with a rheostat and an accuurate Weston ammeter viewed through a low power microscope. For four values of magnet current between 2 to 5 amperes passed for short intervals only, the heating of the magnet core was inappreciable. The torsion angle measured to 0.1 of a degree is proportional to the magnetic field H, which can then be calculated. The experiment was repeated with current in the coil reversed and also with magnetic field reversed and the mean of all four sets taken. This eliminates to a large extent any slight accidental asymmetries in the construction of the coil, in suspension, the initial and final balancing positions any any distortions of the magnetic fields. The experiment was further repeated with three different values of coil current for each of the four values of the magnetic field to observe any systematic error in the current measurement, in the torsion measurement or due to heating effect in the coil. After every reading the initial position of the coil was checked which remained perfectly steady with moderate torsions used by us. The values of the fields were compared with those obtained by the ballistic method using a search coil of approximately the same size as the suspended coil and also by the crystal anisotropy method, already mentioned using $CuSO_4.5H_2O$ crystal suspended with the c-axis vertical (Krishnan and Mukherjee, 1938).

EXPERIMENTAL RESULTS

Torsion Constant of the Fibre.

For standard glass vibrator :

mass, $M = .8280$ gms. $\pm .0001$
radius $r = .9980$ cms. $\pm .0001$
thickness $= .1067$ cms $\pm .0001$

Moment of inertia $= I = M/4\left(r^2 + \frac{t^2}{3}\right)$

$= .2088$ c.g.s. $\pm .0001$

Mean time period

$T = 5.384 \pm .002$

$T_0 = T\left(1 - \frac{\lambda^2}{2\pi^2}\right)$

$= 5.384 \pm .002$

Torsion constant $c = 4\pi^2 \frac{I}{T_0^2}$

$= .2845 \pm .0002$

For the coil :

No. of complete turns of wire, $n = 48$
mean $R^2 = .5258 \pm .0004$
where R = effective radius of the coil.

TABLE I

Values of H

Current in the magnet in amps	Current in the coill in micro amps	θ in degrees (Mean values)	H in Oersteds	Mean for two latter values	H, Search coil values	H, Crystal values
2.2	25.47 ± .01 49.05 ,, 70.49 ,,	70.3 ± .1 135.2 ,, 194.2 ,,	1728 1726 1725	1726 ±1.5	1727 ±5	1727
3.0	25.47 ,, 49.05 ,, 70.49 ,,	86.4 ,, 166.1 ,, 238.4 ,,	2123 2120 2118	2119 ±1.5	2124 ±5	2122
4.0	25.47 ,, 49.05 ,, 70.49 ,,	99.7 ,, 191.6 ,, 275.2 ,,	2450 2447 2446	2447 ±1.5	2450 ±5	2448
5.0	25.47 ,, 49.05 ,, 70.49 ,,	111.6 ,, 214.6 ,, 308.7 ,,	2743 2740 2742	2741 ±1.5	2748 ±5	2748

The values for the residual fields have been measured after steady magnetic state has been obtained in the usual manner for different coil currents given in the table and come out as 74 ± 2.5, 87 ± 0.9 respectively. We may then take the mean of the last two values as most probable and 86 ± 1.1 Oersteds. The corresponding search coil value is 55 Oersteds. The present method is thus much superior to the latter method for small fields.

It is evident from Table I that the first value of the magnetic field corresponding to the lowest coil current is not very accurate since the angle of torsion is rather low. The latter two values are individually more reliable and compare well with each other. So the mean of these two have been taken as the most representative of the values of the fields. Use of higher coil currents leads to difficulties in measurement due to heating effect, distortions in the field, yielding of the silver fibre, etc. The values of the fields obtained are estimated to be accurate to within about 8 to 10 parts in 10,000. Agreement with other methods are good though these are less accurate. The error in the crystal method is not exactly known for the anisotropy value is taken from Krishnan and Mukherjee's paper in which field was measured by a search coil and fluxmeter but the accuracy of measurement was not mentioned.

ACKNOWLEDGMENTS

The author expresses his sincerest thanks to Dr. A. Bose, D.Sc. for suggesting the problem and for his keen interest throughout the progress of the work. Thanks are also due to Sri A. K Dutta and Sri S. Datta for their kind help.

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